

K^0 meson physics in the gravitation field: a constraint on the equivalence principle

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Abstract

$K^0 - \bar{K}^0$ oscillations are extremely sensitive to the K^0 and \bar{K}^0 energy at rest. Even assuming $m_{K^0} = m_{\bar{K}^0}$, the energy is not granted to be the same if gravitational effects on K^0 and \bar{K}^0 slightly differ. We consider various gravitation fields present and, in particular, galactic fields, which provide a negligible acceleration, but relatively large gravitational potential energy. A constraint from a possible effect of this potential energy on the kaon oscillations is found to be

$$\left| \left(\frac{m_g}{m_i} \right)_{K^0} - \left(\frac{m_g}{m_i} \right)_{\bar{K}^0} \right| \leq 8 \times 10^{-13} \quad \text{at CL=90\%}.$$

The derived constraint is competitive with other tests of universality of the free fall. Other applications are also discussed.

Introduction

General relativity suggests the equivalence principle, which means that the gravitational mass is equal to the inertial one, or, which is the same, that the acceleration of free fall is universal. A number of tests for the universality of the free fall acceleration have reached an impressive level of below a part in 10^{12} . In particular, a laboratory test was performed on beryllium and titanium test bodies in the Earth gravitational field [1]

$$\eta_{\text{Be-Ti}} = (0.3 \pm 1.8) \times 10^{-13}, \quad (1)$$

and evaluation of the Lunar Laser Ranging data delivered a constraint on the Earth-Moon system in the Sun field [2]

$$\tilde{\eta}_{\text{E-M}} = (-1.0 \pm 1.4) \times 10^{-13}, \quad (2)$$

where

$$\begin{aligned} \eta_{ab} &= \frac{2(g_a - g_b)}{g_a + g_b}, \\ \tilde{\eta}_{ab} &= \left(\frac{m_g}{m_i} \right)_a - \left(\frac{m_g}{m_i} \right)_b, \end{aligned} \quad (3)$$

$g_{a,b}$ is the acceleration of free fall of the body a or b towards the Earth or the Sun, and m_i and m_g are the inertial and gravitational masses, correspondingly, of the probe body a or b .

Parameters η_{ab} and $\tilde{\eta}$ are very similar to each other and parameterize a deviation from the equivalence principle. Under certain assumptions they coincide.

Most of the paper is devoted to the system $K^0-\bar{K}^0$ in the galactic gravitational field, while in the conclusions we consider various other probe particles and existing fields. Eventually we derive a constraint on the value of $\tilde{\eta}_{K^0\bar{K}^0}$.

Still, there is a certain difference in its interpretation comparing with a traditional constraint for η and $\tilde{\eta}$. Usually, we know from independent experiments that $m_g \approx m_i$. Such a statement for kaons may be obtained as a result of model-dependent indirect interpretation of existing data.

Basics

Our consideration includes the following elements

- we assume that in the absence of gravity there is CPT symmetry and $m_{K^0} = m_{\bar{K}^0}$ (looking for any exotic physics, such as possible violation of the universality of the free fall one cannot blindly rely on an established paradigm, but rather state explicitly, which theoretical suggestions and experimental data are involved);
- we suggest that the appropriate approximation for gravitation-induced corrections to the $K^0-\bar{K}^0$ oscillations is consideration at zero velocity;
- we suggest that the two ‘local’ established components of motion of the Solar system with velocities of 259 km/s and 185 km/s [3] can be treated as a kind of circular motion on a Newtonian orbit.

The idea of our constraint is very simple. Experimental consideration of the $K^0-\bar{K}^0$ oscillations sets a constraint on the value of (see, e.g., [4, 5])

$$\frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{K^0}} \leq 0.8 \times 10^{-18} \quad \text{at CL=90\% ,} \quad (4)$$

which is valid, however, once we assume the equivalence principle. In general, the quantity in the left-hand side of the inequality is rather to be substituted for

$$\frac{|E_K^{(0)} - E_{\bar{K}^0}^{(0)}|}{m_K c^2} , \quad (5)$$

where $E^{(0)}$ is the energy at rest, which includes the gravitation energy. We believe that for slow-meson experiments a consideration of oscillations at rest is a good approximation which takes into account a main effect of gravity within or beyond the equivalence principle.

As long as the equivalence principle is assumed, the gravitation effects for kaon and antikaon cancel in the numerator and, as long as CPT symmetry is assumed, the non-gravitational effects also cancel. Otherwise we obtain:

$$\frac{|E_K^{(0)} - E_{\bar{K}^0}^{(0)}|}{m_K c^2} = \left| \left(\frac{m_g}{m_i} \right)_{K^0} - \left(\frac{m_g}{m_i} \right)_{\bar{K}^0} \right| \left| \frac{U(r) - U(\infty)}{c^2} \right| , \quad (6)$$

where because of CPT invariance we assume that at $r = \infty$ the numerator is equal to zero and that its deviation from zero can only be caused by the non-universality in the gravitational interaction, which in particular may be not the same for K^0 and \bar{K}^0 .

Galactic gravitation fields

The motion of the Solar system in respect to the frame where the cosmic microwave background radiation is isotropic has at least three components and two of them with velocities 259 km/s and 185 km/s can be identified [3] as related to rotation around certain galactic-scale attractors.

For a rotation around a center, which is free fall, we can write for gravitational acceleration

$$\begin{aligned} a &= -\frac{U(r)}{r} \\ &= \frac{\mathbf{v}^2}{r} \end{aligned} \quad (7)$$

and find the potential $U(r)$, defined so that $U(r = \infty) = 0$. In this case, we need to know only the velocity of rotation

$$U(r) = -\mathbf{v}^2 .$$

If we have a few attractors, then

$$U_{\text{tot}} = \sum_i U_i .$$

Summing the two well-understood rotating components mentioned above [3] we arrive at

$$-\frac{U_{\text{tot}}}{c^2} = \frac{\sum_i \mathbf{v}_i^2}{c^2} \simeq 10^{-6} ,$$

as follows from their velocities.

That leads us to a constraint on the equivalence of the free fall acceleration for kaon and antikaon

$$|\tilde{\eta}_{K^0 \overline{K^0}}| \leq 8 \times 10^{-13} \quad \text{at CL}=90\% , \quad (8)$$

which is compatible with other tests [1, 2].

We have ignored the largest velocity component of 455 km/s [3] since it is unclear whether it may be attributed to a certain rotation. However, any additional rotational component will only increase U_{tot} and thus will make the constraint even stronger.

This test is very specific for three reasons:

- It includes a theoretical assumption (such as CPT invariance: $m_{K^0} = m_{\overline{K^0}}$).
- The probe particles are not baryons.
- The field source is not purely baryonic and includes a portion of the dark matter.

Meanwhile, the other precision tests [1, 2] do not need CPT invariance for evaluation of the data, they are performed either on bulk objects or on atomic beams in a field from a pure baryonic-matter source.

Other gravitational sources and other probe particles

One can perform less accurate tests studying $D^0 - \overline{D^0}$ and $B^0 - \overline{B^0}$ oscillations. From [4] we estimate

$$\begin{aligned} \frac{|m_{D^0} - m_{\overline{D^0}}|}{m_{D^0}} &\leq 10^{-15} , \\ \frac{|m_{B^0} - m_{\overline{B^0}}|}{m_{B^0}} &\leq 10^{-12} . \end{aligned}$$

That sets still strong constraints on some exotic components of gravity at the level of

$$|\tilde{\eta}_{D^0\overline{D}^0}| \leq 10^{-9},$$

$$|\tilde{\eta}_{B^0\overline{B}^0}| \leq 10^{-6}.$$

If one considers the galactic gravitational attractors as not well established, we may consider the rotation of Earth in respect to the Sun with the velocity of 30 km/s which means

$$\frac{U_{\text{Sun}}(r_\infty) - U_{\text{Sun}}(r_{\text{Earth}})}{c^2} \simeq 10^{-8}.$$

That makes all constraints two orders of magnitude weaker.

Antigravity for antiparticle

As a kind of ultimate violation of the equivalence principle, which may also involve CPT violation, there is a suggestion for antigravity of antimatter. In this case we can consider for these experiments a completely different approach to field sources and we can also abandon an *a priori* assumption that the masses of neutral mesons and their antiparticles are the same.

We note that the difference in the Sun-Earth distance for the perihelion and aphelion of the Earth orbit is sufficient to maintain an observable potential difference

$$\frac{U_{\text{Sun}}(r_{\text{perihelion}}) - U_{\text{Sun}}(r_{\text{aphelion}})}{c^2} \simeq 3.2 \times 10^{-10}, \quad (9)$$

which allows to rule out the antigravity for all three kinds of neutral mesons considered above. In principle, mesons consist of a quark and an antiquark ($K^0 = \bar{s}d$, $D^0 = \bar{c}\bar{u}$, $B^0 = \bar{b}\bar{d}$), however, the difference in masses of the quark and antiquark is quite large and any model-dependent speculation on this issue can reduce the sensitivity to antigravity by less than an order of magnitude.

The sensitivity suppression factor for a $\bar{q}_1 q_2$ meson with mass M can be estimated from the masses of current quarks [4] as

$$\kappa_{\bar{q}_1 q_2} = \frac{|m_{\bar{q}_1} - m_{q_2}|}{m_M} \simeq \begin{cases} 0.2, & \text{for } K \\ 0.6, & \text{for } D \\ 0.85, & \text{for } B \end{cases}. \quad (10)$$

Another opportunity is a comparison of day's and night's results. They may be different because of a different distance to the Sun. The difference of potentials ranges from $\Delta U/c^2 \simeq 4.2 \times 10^{-13}$ to zero depending on the latitude. This level of accuracy should clearly rule out antigravity from data on kaons and D -mesons. The accuracy for B -mesons is at about a limit to recognize whether the antigravity is possible.

Indeed, it may happen that experiments on the meson oscillations do not allow a simple interpretation because it may be hard to understand from the published results when exactly the data were taken. However, in such a case the [anti]gravitational effects should produce a ‘noise’ at the level by a number of orders of magnitude above the sensitivity and it should be seen.

One more option, which should work successfully for kaons and maybe for D mesons, is a comparison of data taken from different laboratories. If their heights are different by 100

meters, the difference in potentials is

$$\frac{\Delta U(\Delta r = 100 \text{ m})}{c^2} \simeq 3.2 \times 10^{-14}. \quad (11)$$

Conclusions

We have demonstrated that a study of oscillations of neutral kaons provides us with a precision test of the equivalence principle which is different from other tests assuming CPT invariance and dealing with specific gravitational sources and probe particles.

Effects, uncertainties, sensitivities

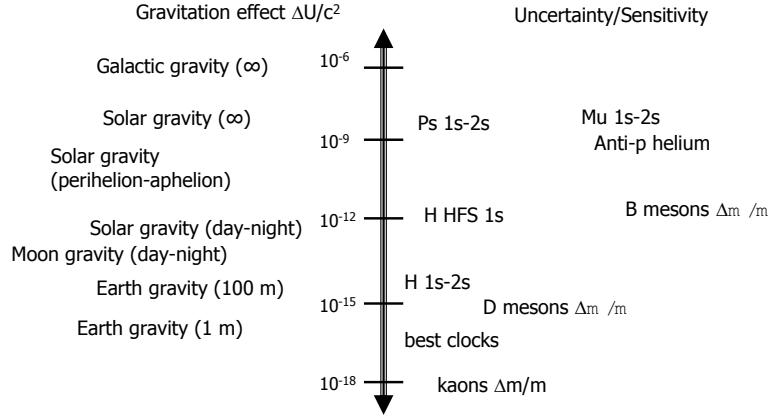


Figure 1: Fractional values of gravitational effects versus uncertainty and sensitivity of various precision measurements (see also [6]).

There are a number of experiments with low uncertainty or with a high sensitivity and a summary is presented in Fig. 1. It happens that kaon physics cannot only provide the best constraint on CPT invariance, but also offers a very sensitive tool to constrain many other exotic options, such as, e.g., a departure from the equivalence principle.

We have also proved that with no additional assumptions antigravity should likely be in conflict with data on oscillations of neutral mesons even in the gravitation field of the Sun and the Earth. More accurate interpretation of the experiments may be needed for a rigorous statement.

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